DART XIII Program

Day: May 26, 2025 Monday **Room: N204**

Time	Speaker	Title
9:00-9:20		Registration
9:20-9:30		Open ceremony
Time	**Speaker**	**Title**
9:30 - 10:20	Joris van der Hoeven	Effective elimination for D-algebraic power series equations
10:30 - 11:00	Tea break	
11:00 - 11:50	Frederic Chyzak	First-order factors of linear Mahler operators
12:00 - 14:00	Lunch at Wuke Hotel	
14:00 - 14:50	Georg Regensburger	Integro-differential rings with generalized evaluation
15:00 - 15:50	Khalil Ghorbal	On the existence of principal differential ideals for polynomial derivations: recent advances and computational aspects
16:00 - 16:30	Tea break	
16:30 - 17:20	Shiva Shankar	Discrete systems on $\mathbb{Z}^n$ and their symmetries

Day: May 27, 2025 Tuesday **Room: N204**

Time	Speaker	Title
9:00 - 9:50	Françoise Ollivier	Bézout identities and control of the heat equation
10:00 - 10:30	Tea break	
10:30 - 11:20	Françoise Point	On definable groups and D-groups in certain fields with a generic derivation and their algebraic properties
11:40 - 14:00	Lunch at Wuke Hotel	
14:00 - 14:50	Max Tschaikowski	Algorithmic model reduction across disciplines
15:00 - 15:50	Chengming Bai	Deformation families of Novikov bialgebras via differential antisymmetric infinitesimal bialgebras
16:00 - 16:30	Tea break	
16:30-17:20	Xing Gao	Rough differential equations and rough path theory
18:00-21:00		Banquet at Wuke Hotel

Day: May 28, 2025 Wednesday **Room: N204**

Time	Speaker	Title
9:00 - 9:50	Thomas Scanlon	Minimality of difference-differential equations
10:00 - 10:30	Tea break	
10:30 - 11:20	Ivan Tomasic	Galois theory of differential and difference schemes
11:40 - 14:00	Lunch at Wuke Hotel	
14:00-18:00		Free discussions

Day: May 29, 2025 Thursday **Room: N219**

Time	Speaker	Title
9:00 - 9:50	Michael Schlosser	Elliptic partial fraction decompositions and (multivariate) elliptic hypergeometric series
10:00 - 10:30	Tea break	
10:30 - 11:20	Bin Zhang	Extended shuffle product for multiple zeta values
11:40 - 14:00	Lunch at Wuke Hotel	
14:00 - 14:50	Daniel Robertz	Normal forms in differential Galois theory for the classical groups
15:00 - 15:50	Colin Faverjon	Purity theorems and transcendence: the case of linear Mahler equations
16:00 - 16:30	Tea break	
16:30-17:20	Alexander Demin	Computing globally identifiable parametrizations of dynamical models

Day: May 30, 2025 Friday **Room: N219**

Time	Speaker	Title
9:00 - 9:50	Sonia Rueda	Computing algebro-geometric ODOs and their spectral curves
10:00 - 10:30	Tea break	
10:30 - 11:20	Robert Dougherty-Bliss	Enumerating balanced matrices
11:40 - 14:00	Lunch at Wuke Hotel	
14:00 - 14:50	Seda Albayrak	A refinement of Christol's theorem for sparse algebraic power series
15:00 - 15:50	Rong-hua Wang	Power-partible reduction and congruences of the Delannoy polynomials
16:00 - 16:30	Tea break	
16:30-17:20	Ziming Li	Complete reduction for integration in finite terms

Abstracts

A refinement of Christol's theorem for sparse algebraic power series

Seda Albayrak

Simon Fraser University
Canada

Abstract

A famous result of Christol gives that a power series $F(t) = \sum_{n\geq 0} f(n)t^n$ with coefficients in a finite field $\mathbb{F}_q$ of characteristic p is algebraic over the field of rational functions in t if and only if there is a finite-state automaton accepting the base-p digits of n as input and giving $f(n)$ as output for every $n \geq 0$. An extension of Christol's theorem, giving a complete description of the algebraic closure of $\mathbb{F}_q(t)$, was later given by Kedlaya. When one looks at the support of an algebraic power series, that is the set of n for which $f(n) \neq 0$, a well-known dichotomy for sets generated by finite-state automata shows that the support set is either sparse—with the number of $n \leq x$ for which $f(n) \neq 0$ bounded by a polynomial in $\log(x)$—or it is reasonably large in the sense that the number of $n \leq x$ with $f(n) \neq 0$ grows faster than x^α for some positive α. The collection of algebraic power series with sparse supports forms a ring and we give a purely algebraic characterization of this ring in terms of Artin-Schreier extensions and we extend this to the context of Kedlaya's work on generalized power series. (Joint work with Jason Bell).

Deformation families of Novikov bialgebras via differential antisymmetric infinitesimal bialgebras

Chengming Bai

Nankai University
China

Abstract

We generalize S. Gelfand's classical construction of a Novikov algebra from a commutative differential algebra to get a deformation family $(A, \circ_q)$ of Novikov algebras by an admissible commutative differential algebra, which ensures a bialgebra theory of commutative differential algebras, enriching the antisymmetric infinitesimal bialgebra. Moreover, a deformation family of Novikov bialgebras is obtained, under certain further condition. In particular, we obtain a bialgebra variation of S. Gelfand's construction with an interesting twist: every commutative and cocommutative differential antisymmetric infinitesimal bialgebra gives rise to a Novikov bialgebra whose underlying Novikov algebra is $(A, \circ_{-\frac{1}{2}})$ instead of $(A, \circ_0)$ which recovers the construction of S. Gelfand. This is the joint work with Yanyong Hong and Li Guo.

First-order factors of linear Mahler operators

Frederic Chyzak

INRIA
France

Abstract

We discuss two algorithms for computing first-order right-hand factors in the ring of linear Mahler operators $\ell_r M^r + \cdots + \ell_1 M + \ell_0$ where $\ell_0, \ldots, \ell_r$ are polynomials in x and $Mx = x^b M$ for some integer $b \geq 2$. In other words, we consider algorithms for finding all formal infinite product solutions of linear functional equations $\ell_r(x) f(x^{b^r}) + \cdots + \ell_1(x) f(x^b) + \ell_0(x) f(x) = 0$. We will just sketch the first of our algorithms, which is adapted from Petkovšek's classical algorithm for the analogous problem in the case of linear recurrences. We will mainly focus on the second of our algorithms, which proceeds by computing a basis of generalized power series solutions of the functional equation and by using Hermite–Padé approximants to detect those linear combinations of the solutions that correspond to first-order factors.
Implementations of both algorithms are available, and we will discuss their use in combination with criteria from the literature to prove the differential transcendence of power series solutions of Mahler equations.

Computing globally identifiable parametrizations of dynamical models

Alexander Demin

National Research University HSE
Russia

Abstract

Dynamical models described by ordinary differential equations (ODEs) are a fundamental tool in modeling. Structural identifiability is a property of a dynamical model that determines if the model parameters and states can be identified from the measured data under ideal conditions. In this talk, we will present an algorithm for finding globally identifiable parametrizations of a model described by a system of ODEs, where each state and parameter in the new parametrization is directly related to the original states and parameters. Additionally, we will demonstrate the implementation of the algorithm in the Julia package StructuralIdentiafiability.jl.
This work is a joint result by Alexander Demin, Gleb Pogudin, and Chris Rackauckas.

Enumerating Balanced Matrices

Robert Dougherty-Bliss

Dartmouth College
USA

Abstract

A $\{0, 1\}$-matrix is balanced if it has as many 1's as 0's in every row and column. For fixed k, the number of $2n \times 2k$ balanced matrices is D-finite in n, but it is difficult to find enumeration formulas or compute explicit recurrences. I will show a formula for $k = 1$, a rigorous recurrence for $k = 2$ and $k = 3$, and a conjectured recurrence for $k = 4$. Joint work with Christoph Koutschan, Natalya Ter-Saakov, and Doron Zeilberger.

Purity theorems and transcendence: the case of linear Mahler equations

Colin Faverjon

Université de Lyon 1
France

Abstract

A 'Purity Theorem' is a statement asserting that certain properties of solutions to functional equations extend to all solutions of their minimal equations. Twenty-five years ago, André demonstrated how the celebrated Siegel-Shidlovskii Theorem in transcendental number theory can be derived from a purity theorem for solutions of linear differential equations.
In a recent work with Julien Roques, we have established that similar purity phenomena occur in the framework of linear Mahler equations. These equations naturally arise in the theory of finite automata and in problems related to numeration systems. Our results imply, among other findings, that the minimal Mahler equation of the generating series of an automatic sequence possesses a basis of solutions that 'emerge from the automatic world'. We will explore the connections between these results and the transcendental theory of values of Mahler functions.

Rough differential equations and rough path theory

Xing Gao

Lanzhou University
China

Abstract

Rough paths were introduced by T. Lyons in 1998 as a new tool for solving rough differential equations:

$$\begin{cases} dY_t &= F(Y_t)\cdot dX_t = \sum_{i=1}^{d} F^i(Y_t)dX_t^i, \quad \forall t\in[0,T], \\ Y_0 &= \xi, \end{cases}$$

They are built from substitutes of Chen's iterated integrals of a path $X:[0,T]\to\mathbb{R}^d$ when this path is α-Hölder-continuous with $\alpha\in(0,1]$. A milestone result of rough paths is that Hairer, 2014 Fields Medal winner, employed rough path theory to solve the KPZ equation, obtaining the expected solution of physicists. Later Hairer generalized rough paths to regularity structures, with the goal to solve stochastic partial differential equations.

On the Existence of Principal Differential Ideals for Polynomial Derivations: Recent Advances and Computational Aspects

Khalil Ghorbal

Inria-Rennes
France

Abstract

We restrict ourselves to the (seemingly) simple settings of polynomial derivations with a focus on the long standing open problems pertaining to the existence of principle differential ideals and their concise representation. Those particular ideals play an important theoretical role in the integrability of polynomial dynamical systems and have found several applications (e.g. theoretical physics) since their introduction in the seminal work of Gaston Darboux, almost 150 years ago. We show that the coefficients of the (unique) generator have to satisfy underlying recurrences that can be made explicit. This key insight shed some light on the structure of such objects: not only it allows to reason about their existence, it also suggest a completely novel approach for their effective computation.

Complete Reduction for Integration in Finite Terms

Ziming Li

AMSS, Chinese Academy of Sciences
China

Abstract

Let V be a linear space and U a subspace of V. A linear idempotent on V is called a *complete reduction* for U if $U = \ker(\phi)$. Let F be a differential field with the constant subfield C, and F' denote the set of derivatives in F. Then F' is a linear subspace over C. A complete reduction ϕ for F' on F allows us to decompose an element f of F as the sum of a derivative and the remainder $\phi(f)$. A direct application of ϕ is that $f \in F'$ if and only if $\phi(f) = 0$.

The talk consists of two parts. First, a complete reduction for F' on F is presented algorithmically when F is a primitive tower. An article based on this work has been accepted by the 2025 ISSAC conference. Next, we outline results obtained from an on-going project for developing complete reductions on transcendental Liouvillian extensions, in which the complete reductions are not only for derivatives but also for images of Risch operators. Let $F = C(x)(t_1, \ldots, t_n)$ be a differential field extension over $(C(x), d/dx)$, where C is a field of characteristic zero. Assume further that $t_1, \ldots, t_n$ are algebraically independent over $C(x)$ and that C is also the subfield of constants in F. Then F is called a *primitive tower* over $C(x)$ if $t_i' \in F_{i-1}$, and it is called a *transcendental Liouvillian extension* if either $t_i' \in F_{i-1}$ or $t_i'/t_i \in F_{i-1}$, where $i \in \{1, \ldots, n\}$ with $F_0 = C(x)$ and $F_i = F_{i-1}(t_i)$. Typical examples for primitive towers are differential fields generated by (poly-)logarithmic functions and the logarithmic integral, while transcendental Liouvillian extensions take primitive towers as a special case and may have hyperexponential functions, the exponential integral and error function as generators. Complete reductions lead to alternative algorithms for determining whether an element in a primitive tower or a transcendental Liouvillian extension has an elementary integral, and computing such an integral when there exists one.

This is joint work with Shaoshi Chen, Hao Du, Yiman Gao, Hui Huang and Wenqiao Li.

Bézout identities and control of the heat equation

Francois Ollivier

Ecole Polytechnique
France

Abstract

Computing analytic Bézout identities remains a difficult task, which has many applications in control theory. Flat PDE systems have cast a new light on this problem. We consider here a simple case of special interest: a rod of length $a + b$, insulated at both ends and heated at point $x = a$. The case $a = 0$ is classical, the temperature of the other end $\theta(b, t)$ being then a flat output, with parametrization $\theta(x, t) = \cosh((b - x)(\partial/\partial t)^{1/2}\theta(b, t)$.

When a and b are integers, with a odd and b even, the system is flat and the flat output is obtained from the Bézout identity $f(x)\cosh(ax) + g(x)\cosh(bx) = 1$, the computation of which boils down to a Bézout identity of Chebyshev polynomials. But this form is not the most efficient and a smaller expression $f(x) = \sum_{k=1}^{n} c_k \cosh(kx)$ may be computed in linear time.

These results are compared with an approximations by a finite system, using a classical discretization.

We provide experimental computations, approximating a non rational value r by a sequence of fractions b/a, showing that the power series for the Bézout relation seems to converge.

On definable groups and D-groups in certain fields with a derivation and their algebraic properties

Francoise Point

Paris Diderot University
France

Abstract

Let T be a complete, model-complete geometric L-theory of fields of characteristic 0 and let T(∂) be the theory of expansions of models of T by a derivation ∂. We assume that T(∂) has a model-companion T(∂) and satisfies further assumptions that hold in the following cases: T is a model-complete theory of a large pure field of characteristic 0 (possibly with additional constants) [10] (when T is the theory of real-closed fields, T@ is the theory of closed ordered differential fields, described by M. Singer [9]), T is the theory of an algebraically closed valued field of characteristic 0 [4], T is the theory of an henselian field of characteristic 0 [2], or T is the theory of an o-minimal expansion of the field of real numbers where the derivation ∂ respects the definable functions, as described in [3]. In joint work with K. Peterzil and A. Pillay [5], we proved that any finite-dimensional group definable in models of T(∂) definably embeds in a definable group G in a model of T, using a variant of Weil's pre-group construction [11]; further in certain cases, we endowed G with a structure of a definable D-group [6], generalizing the classical construction of Buium of algebraic D-groups [1]. In this talk we will first recall those results and we will relate the algebraic properties of T and G. We will also show how to extend some of the above results for certain topological fields of characteristic 0 with either a dp-minimal theory [8] or an open theory as defined in [2].

References.

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Integro-differential rings with generalized evaluation

Georg Regensburger

University of Kassel
Germany

Abstract

In this talk, we discuss the fundamental theorem of calculus and its algebraic implications in differential rings, focusing on functions with singularities and a generalized notion of evaluation. We give an overview of integro-differential rings and present several examples. This approach generalizes results such as shuffle relations for nested integrals and the Taylor formula, incorporating additional terms to account for singularities.
In general, not every element of a differential ring has an antiderivative in the same ring. Starting from a commutative differential ring and a direct decomposition into integrable and non-integrable elements, we outline the construction of the free integro-differential ring. This integro-differential closure contains, in particular, all nested integrals over elements of the original differential ring.
This talk is based on joint work with Clemens Raab.

Normal Forms in Differential Galois Theory for the Classical Groups

Daniel Robertz

RWTH Aachen University
Germany

Abstract

Let G be a classical group of dimension d and let $\boldsymbol{a} = (a_1, \ldots, a_d)$ be differential indeterminates over a differential field F of characteristic zero with algebraically closed field of constants C. Further let $A(\boldsymbol{a})$ be a generic element in the Lie algebra $\mathfrak{g}(F\langle \boldsymbol{a} \rangle)$ of G obtained from parametrizing a basis of $\mathfrak{g}$ with the indeterminates $\boldsymbol{a}$. It is known that the differential Galois group of $\boldsymbol{y}' = A(\boldsymbol{a})\boldsymbol{y}$ over $F\langle \boldsymbol{a} \rangle$ is $G(C)$.
In this talk we report on recent joint work with Matthias Seiss. in which we constructed a differential field extension $\mathcal{L}$ of $F\langle \boldsymbol{a} \rangle$ such that the field of constants of $\mathcal{L}$ is C, the differential Galois group of $\boldsymbol{y}' = A(\boldsymbol{a})\boldsymbol{y}$ over $\mathcal{L}$ is still the full group $G(C)$ and $A(\boldsymbol{a})$ is gauge equivalent over $\mathcal{L}$ to a matrix in normal form introduced by Seiss. We also consider specializations of the coefficients of $A(\boldsymbol{a})$ as well as determining the differential Galois group of the specialized equation.

Computing algebro-geometric ODOs and their spectral curves

Sonia Rueda

Universidad Politécnica de Madrid
Spain

Abstract

The correspondence between commutative rings of ordinary differential operators and algebraic curves has been extensively and deeply studied since the seminal works of Burchnall-Chaundy in 1923. In the last decade, we worked on making this correspondence computationally effective, by means of differential algebra and symbolic computation.
The centralizer of an ordinary differential operator (ODO) L is a maximal commutative ring, the affine ring of a spectral curve Γ, and contains the ring of polynomials $C[L]$, with coefficients in the field of constants C of the coefficient field K of L. We present an algorithm to compute a finite basis of $\mathcal{Z}(L)$ as a $C[L]$-module. Our approach combines results by K. Goodearl in 1985 with solving systems of equations of the stationary Gelfand-Dickey hierarchy (sGD). In addition, by considering parametric coefficients, we generate families of ODOs with non trivial centralizer, in particular algebro-geometric, whose coefficients are solutions in K of systems of the sGD hierarchy.
The basis of the centralizer allows to compute the defining ideal of Γ by means of differential resultants. In addition, the factorization problem of $L - \lambda$ over a new coefficient field, defined by K and Γ, allows the computation of the spectral sheave on Γ. The ultimate goal is to develop Picard-Vessiot theory for spectral problems $L(y) = \lambda y$, for a generic λ.

This project is developed in collaboration with M.A. Zurro, A. Jimenez Pastor, R. Hernandez-Heredero and R. Delgado under the research grant "Algorithmic Differential Algebra and Integrability" (ADAI), PID2021-124473NB-I00.
`https://sites.google.com/view/sonialrueda/proyecto-adai`.

Minimality of difference-differential equations

Thomas Scanlon

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USA

Abstract

We analyze the behavior of systems of algebraic differential equations when considered as systems of difference-differential equations, with special emphasis on systems which define strongly minimal sets relative to the theory $\mathrm{DCF}_{0,n}$ of differentially closed fields of characteristic zero with n distinguished commuting derivations. We show that if X is a strongly minimal set relative to $\mathrm{DCF}_{0,n}$ defined by a finite system of algebraic partial differential equations and the forking geometry on X is trivial (also called "disintegrated"), then X remains minimal when regarded as definable set relative to the theory $\mathrm{DCFA}_{0,n}$ of difference-differentially closed fields of characteristic zero with n commuting derivations. We illustrate this theorem by describing the possible difference-differential equations consistent with differential equations of the form $y' = f(y)$ where f is a monic cubic polyomial over differential constants. This is a report on joint work with Wei Li.

Elliptic partial fraction decompositions and (multivariate) elliptic hypergeometric series

Michael Schlosser

University of Vienna
Austria

Abstract

Hypergeometric series are central objects in the theory of special functions and are amenable to algorithmic treatment. There is actually a three-level hierarchy of hypergeometric series, consisting of the families of ordinary, basic, and elliptic hypergeomeric series. While algorithms for the study of ordinary hypergeometric series and for basic hypergeometric series exist and have even been implemented in computer algebra programs, the algorithmic theory for elliptic hypergeometric series is much less developed. This is even more true for multivariate elliptic hypergeometric series such as elliptic hypergeometric multi-sums that can be associated with root systems. In my talk I will survey the different types of hypergeometric series and identities satisfied by them and will in particular highlight the role of elliptic partial fraction decompositions played in the derivation of results (such as explicit summations) for (multivariate) elliptic hypergeometric series.

Discrete systems on $\mathbb{Z}^n$, and their symmetries

Shiva Shankar

Indian Institute of Technology Bombay
India

Abstract

This talk will consider discrete systems on the lattice $\mathbb{Z}^n$ defined by linear difference equations (i.e. by elements of the Laurent polynomial ring) that are invariant under a finite group of symmetries. It will show that there always exist solutions to such systems that are also invariant under this group of symmetries.

Galois theory of differential and difference schemes

Ivan Tomasic

Queen Mary University of London
UK

Abstract

Classical Picard-Vessiot theory had been developed as the Galois theory of differential/difference field extensions associated to linear differential/difference equations. Inspired by categorical Galois theory of Janelidze, and by using novel methods of precategorical descent applied to algebraic-geometric situations, we develop a Galois theory that applies to morphisms of differential/difference schemes, and vastly generalises the linear Picard-Vessiot theory, as well as the strongly normal theory of Kolchin. The talk will be based on a joint paper with Behrang Noohi.

Algorithmic Model Reduction Across Disciplines

Max Tschaikowski

Aalborg University
Denmark

Abstract

Model reductions project an original model to a smaller one while preserving properties of interest, thus allowing to reduce simulation and analysis times. Network embeddings, for instance, allow to project networks onto lower-dimensional spaces while preserving similarities among nodes, while reductions of quantum circuits compress the latter while preserving quantum computation results like minimal ground states. Likewise, differential equivalence achieves preserve biological measurements while reducing the number of chemical reactions.
We will give an overview of algorithmic model reductions that arise from linear invariant subspaces. The first part will focus on graph-based, while the second part on geometry-based approaches. The applicability will be demonstrated on benchmarks from network science, systems biology and quantum computing.

Effective elimination for D-algebraic power series equations

Joris van der Hoeven

Ecole Polytechnique
France

Abstract

The set of differentially algebraic power series in several variables over an effective field of characteristic zero forms an effective "tribe": it is closed under the effective ring operations, restricted division, composition, the implicit function theorem, as well as restricted monomial transformations with rational exponents.
Given an effective tribe with an effective zero test, we shall prove an effective version of the Weierstrass division theorem and show how to use this for the development of an effective elimination theory for power series equations, given through elements in the tribe.

Power-partible reduction and congruences of the Delannoy polynomials

Rong-Hua Wang

Tiangong University
China

Abstract

In this talk, we apply the power-partible reduction to study arithmetic properties of sums involving Delannoy polynomials $D_k(z)$. New families of congruences for $D_k(z)$ will be obtained. We also confirm and generalize a conjecture of V.J.W. Guo and J. Zeng in 2012. This is a joint work with M.X.X. Zhong.

Extended Shuffle product for multiple zeta values

Bin Zhang

Sichuan University
China

Abstract

The shuffle product on positive integer points, which corresponds to the shuffle algebra for multiple zeta values, is extended uniquely to all integer points, by making the linear operator which decreases the first entry by one a differential operator. We then show that all convergent integer points form a subalgebra under this extended shuffle product. By lifting the extended shuffle product to the locality algebra of Chen symbols, we prove that the multiple zeta series defines an algebra homomorphism from the subalgebra of convergent points to real numbers, which shows that the extended shuffle product is a structure for convergent integer points.